Ghostly action at a distance: a non-technical explanation of the Bell inequality

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We give a simple non-mathematical explanation of Bell's inequality. We describe how, when applied to the results of Einstein-Podolsky-Rosen (EPR) experiments (and combined with some auxiliary assumptions), the inequality shows that *some* sort of faster-than-light influence is present in nature. We discuss the implications, emphasizing the distinction between causal influences and signals, the tension between EPR and determinism, and the challenge of reconciling the Principle of Relativity with the EPR results.

I. INTRODUCTION

The Einstein-Podolsky-Rosen (EPR) family of experiments provides a fascinating and famous test of the idea of *locality*. In particular, they test the principle of strong locality, also known as "local causality", which is closely related to the principle that no causal influence can travel faster than light. When EPR-type experiments are performed, the results show that in nature there *are* causal influences that travel faster than light: strong locality is violated. The main goal of this paper is to give an extremely simple non-technical explanation of how EPR experiments lead to this striking conclusion.

Since we are forced to give up strong locality, it is natural to look for alternate, weaker, forms of locality, such as the principle that signals cannot travel faster than light ("signal locality"). We describe how signal locality follows from Einstein's theory of relativity, and show how it can be reconciled with EPR results, but only if we accept that nature has an elusive side, a fundamental ungraspability (e.g. some essential randomness or uncontrollability) that prevents the violation of strong locality from leading to faster-than-light signalling.

The ability of EPR experiments to test the principle of strong locality is expressed by Bell's inequality. Our simple non-mathematical explanation of this is in Sec. III and IV. We will show that Bell's inequality simply says that if you plan yes-or-no answers to three questions then on two randomly chosen questions your plan will lead you to give the same answer to both questions at least 1/3 of the time (Fig. 3).

In order to understand the significance of Bell's inequality, we now give an overview of its logical context (Fig. 1). For pedagogical purposes it is natural to present the argument in two stages, which we will call the "EPR argument", and the "Bell argument" although historically the arguments were not presented in exactly this form [1, 2].

We will concentrate on Bohm's variant of EPR, the "EPRB" experiment. This involves pairs of particles, typically a pair of photons in a spin singlet state. The question at hand is: what general types of theories can account for the observed behavior of these particles? Can strongly local theories do the job? Fig. 1 shows the space of theories of such particles. The thick solid (red) rectangle encloses the set of strongly local theories, the ones in which no causal influence can travel faster than light. The upper thin solid (green) rectangle encloses the set of theories that are "deterministic", meaning that the behavior of the particles is fully determined by their internal properties, without any randomness in the laws of physics. As the figure shows, orthodox quantum mechanics (i.e. the form of quantum mechanics taught in conventional physics courses) is not a deterministic theory. This is because in quantum mechanics the result of a measurement of the spin of a photon is not pre-determined by the state of the photon: the result is fundamentally random, with a probability distribution given by the wavefunction of the photon. In Sec. II we give a more detailed account of the meaning of strong locality and determinism.

The EPR argument [3] can be seen as a rallying cry for those who dislike the indeterminism and randomness of quantum mechanics. Using very simple logic (see Secs. III and IV) one can see from the observed behavior of photons in EPRB experiments that if strong locality holds then the photons must have definite internal states that determine their behavior. The EPR argument therefore rules out strongly local theories that are indeterministic (vertical shading in Fig. 1). This sounds like a refutation of quantum mechanics, which is famously an

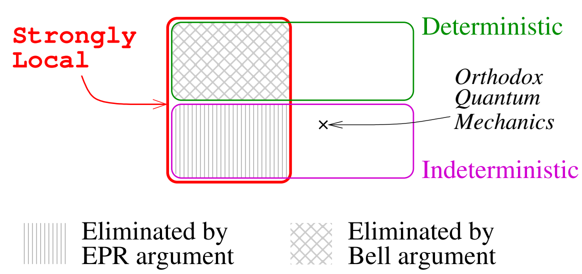

FIG. 1: Venn diagram of the space of theories and the constraints from EPRB experiments. The thick (red) line encloses the set of strongly local theories. The EPR argument concludes that strongly local theories must be deterministic; the Bell argument concludes that strongly local theories cannot be deterministic. In combination, these arguments rule out strongly local theories.

indeterministic theory. However, orthodox quantum mechanics explicitly violates strong locality via the instantaneous collapse of the wavefunction, so EPR's argument does not apply directly to orthodox quantum mechanics. Rather, it shows that any alternate theory that was strongly local would have to be deterministic. In such a theory the result of measuring a photon's spin would not be random; it would be determined by the state of non-quantum-mechanical "hidden variables" that predetermine the behavior of the photon.

The second stage, which we call the "Bell argument" [4, 5], destroys the dream of finding a strongly local and deterministic theory to replace quantum mechanics. Bell pointed out that if nature is described by a strongly local and deterministic theory then the behavior of the photon pairs has to obey a constraint called the "Bell inequality". In Secs. III and IV we will give an elementary explanation of the Bell inequality. We will show that it arises from the fact that if you have planned yes-or-no answers to three questions then on two randomly chosen questions you will give the same answer to both questions at least 1/3 of the time.

In real EPRB experiments the results violate Bell's inequality. This shows that *no* deterministic and strongly local theory can explain the behavior of the photons (cross-hatched shading in Fig. 1). Taken together, the EPR and Bell arguments, combined with the experimental data, show that strong locality must be false. Some causal influences travel faster than light.

The rest of this paper explores the EPR and Bell arguments in as much depth as is possible without mathematical formalism. In Sec. II we lay out in more detail the meaning of the key postulates of strong locality and determinism. In Sec. III we give an intuitive nonmathematical explanation of the Bell inequality and the resultant refutation of strong locality. Sec. III applies these concepts to the real experimental setup involving photon spin measurements. Sec. V looks at alternative forms of locality: since nature does seem to obey some sort of locality principle, we ask what form of locality is compatible with EPRB experiments, and how EPR relates to the Principle of Relativity.

II. LOCALITY, DETERMINISM, AND UNCERTAINTY

The principles that are tested by EPRB experiments are:

1. **Determinism:** The result of any measurement is determined by independently-existing properties of the system that is being measured [1, 6]. Those properties are sometimes phrased as the "state of the hidden variables". In a deterministic theory, even for a measurement that was not actually performed there is a fact of the matter about what result it would have yielded ("counterfactual definiteness").

2. **Strong Locality:** This principle was called "local causality" by Bell in [5], and is sometimes called "factorizability" for reasons we will explain below. Given Reichenbach's principle of common cause, it follows from the idea that causal influences cannot travel faster than light, so the causal influences that affect an event must be in its past light cone (Fig. 2).

We now explain both of these in more detail. Readers interested in getting straight to the EPR and Bell arguments can skip the rest of this section.

Determinism

To get a sense of what is meant by determinism, let's take the example of a particle, such as a cosmic ray, flying though space. According to determinism, the particle has well-defined properties like its mass, speed, and amount of spin, and it has these properties whether or not you do any measurements on it. If you wanted to, you could measure its speed (by bouncing photons off it, for example), but the particle's speed has the value it has, whether or not you measure it. According to determinism, measurements just expose existing properties of the particle.

Determinism is intimately bound up with our understanding of *uncertainty*. One can distinguish two ways in which we may be uncertain about the outcome of a measurement:

1. Uncertainty arising from our ignorance. The outcome of the measurement is encoded in the state of the object, but we don't have sufficiently accurate information about its state to predict what value the measurement will yield.

2. Fundamental uncertainty: the outcome of the measurement is not determined in advance of the measurement; the system doesn't "decide" how to behave until the measurement is actually performed.

In ordinary life, and in science up until the advent of quantum mechanics, all the uncertainty that we encounter is presumed to be of the first kind, uncertainty arising from ignorance. We can't predict the weather very accurately, but the more we learn about the state of Earth's atmosphere and oceans and the laws they obey, the better our predictions become. Determinism says that *all* uncertainty is of the first kind, the kind that arises only from our ignorance. Determinism is a sort of scientific optimism: objects have properties, and by performing measurements we can learn what those properties are; the outcome of every measurement is determined by the state of the measuring apparatus and the object being measured, and if we knew enough about that state we could predict the outcome of the measurement.

Quantum mechanics introduced the idea that there might be uncertainty of the second type: that nature might be fundamentally indeterminate. As we will see in Sec. IV, Einstein, Podolsky, and Rosen then argued

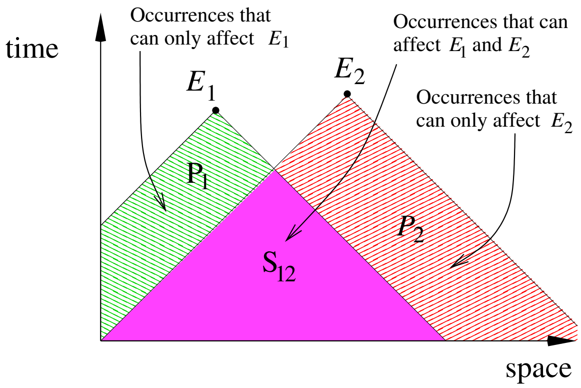

FIG. 2: Strong locality states that any correlation between the results of two measurements E_1 and E_2 made at different spacetime locations must arise from events in the region S_{12} that is in *both* their past light cones.

that uncertainty of this type can lead to conflict with the principle of strong locality.

Strong Locality

As we have mentioned above, strong locality arises from the principle that no causal influence can travel faster than light, combined with the assumption (known as "Reichenbach's principle of common cause") that correlations arise from common causes, and once those have been taken into account the correlation should disappear. In the rest of this paper we will always make this assumption; for a deeper analysis of Reichenbach's principle and the concept of causation in Bell's inequality, see Ref. [7, 8].

Before explaining strong locality in more detail, we note that there are other principles that are related to locality, such as "information cannot be transmitted faster than light" or "there is no preferred inertial reference frame" (the Principle of Relativity). In Sec. V we will discuss how these are related to strong locality and the results of EPRB experiments.

In order to be able to talk about causal influences traveling from place to place we have to assume that events that could act as causes are localized in space. Then we can express strong locality in the following two statements [1, 9]:

An event can only be causally influenced by other events that are localized in its past light cone. (1)

Once all shared causal influences λ on two events E_1 and E_2 have been taken into account, their predicted probabilities factorize, $p(E_1, E_2 \,|\, \lambda) = p(E_1 \,|\, \lambda) \times p(E_2 \,|\, \lambda)$ (2)

This is illustrated in Fig. 2. The region P_1 contains all events that can causally influence E_1 but not E_2. In other words, P_1 is the set of points in spacetime that have E_1 but not E_2 in their forward lightcone. Conversely, P_2 contains all events that can causally influence E_2 but not E_1. Events in the region S_{12} can causally influence both E_1 and E_2.

The strong locality statement ((1) and (2)) says that only in S_{12} can there be events λ that influence both E_1 and E_2, and thereby lead to a correlation between them (i.e., non-factorizability of their joint probability distribution). Anything that happens outside S_{12} cannot cause a correlation between E_1 and E_2, because it cannot affect both of them.

III. EPR AND BELL WITH HUMANS

In order to make the EPR and Bell arguments as comprehensible as possible we now explain them using an analogy where instead of experimenting on photons we are questioning people.

A. Testing twins for supraluminal telepathy

Imagine that someone has told us that twins have special powers, including the ability to communicate with each other using telepathic influences that are "supraluminal", meaning they travel faster than light. We decide to test this by collecting many pairs of twins, separating each pair, and asking them questions to see if their answers agree.

To make things simple, we will only have three possible questions, and they will be Yes/No questions. We will tell the twins in advance what the questions are.

The procedure is as follows.

1. A new pair of twins is brought in and told what the three possible questions are.
2. The twins travel far apart in space to separate questioning locations.
3. At each location there is a questioner who selects one of the three questions at random, and poses that question to the twin in front of her.
4. **Spacelike separation**: When the question is chosen and asked at one location, there is not enough time for any influence travelling at the speed of light to get from there to the other location in time to affect either what question is chosen there, or the answer given.

B. EPR argument for the twins

Now, suppose we perform this experiment and we find **same-question agreement**: whenever both twins happen to get asked the same question, their answers always agree. How could they do this? Here are two possible explanations,

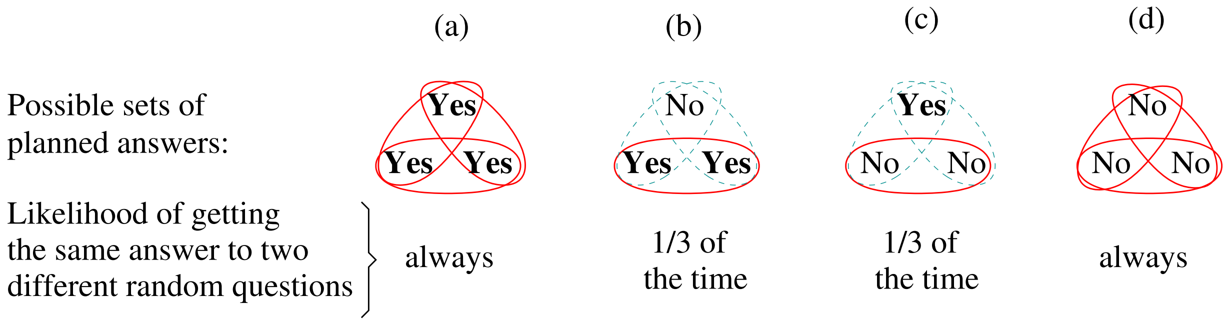

FIG. 3: The essence of the Bell inequality (Eq. (3)). In formulating a plan for how to give Yes/No answers to three questions, there are four types of plan. No matter what plan one follows, the answers to two different randomly chosen questions will be the same *at least 1/3 of the time.*

1. The twins are communicating with each other faster than light in order to make sure their answers agree.

2. The twins are following a *plan*. Before they were separated they agreed in advance what their answers to the three questions would be.

The same-question agreement that we observe does not prove that the twins are communicating telepathically faster than light. If we believe that strong locality is a valid principle, then we can resort to the other explanation, that the twins are following a plan. This inference corresponds to the EPR argument: strong locality (the twins cannot communicate faster than light), when combined with same-question agreement, implies determinism (the twins are following a predefined plan). For the twins to follow a plan means they must each have some definite internal state, their memory of the plan, that determines how they respond to questions. For twins this does not seem so remarkable, but for photons it is a striking claim, because the quantum mechanical picture of a photon does not allow for any such internal state that determines the outcome of measurements on the photon. That is the point of the EPR argument (vertical shading in Fig. 1): if nature obeys strong locality then quantum mechanics is incomplete and there is a deterministic theory that describes nature equally accurately.

C. Bell inequality for the twins

In the thought experiment as described up to this point we only looked at the recorded answers in cases where each twin was asked the same question. There are also recorded data on how they responded when the two questioners happened to choose different questions. Bell noticed that this data can be used as a cross-check on our strong-locality-saving idea that the twins are following a plan. The cross-check takes the form of an inequality:

<u>Bell inequality for twins:</u>

If the twins are following a plan then, when each is asked a *different* randomly chosen question, their answers will be the same, on average, at least 1/3 of the time. (3)

Fig. 3 illustrates why (3) is true. For each pair of twins, there are four general types of plan they could adopt to determine how they would answer each of the three possible questions.
(a) a plan in which all three answers are Yes;
(b) a plan in which there are two Yes and one No;
(c) a plan in which there are two No and one Yes;
(d) a plan in which all three answers are No.
If the twins are both following the same predefined plan, as strong locality and same-question agreement imply, then when the random questioning leads to each of them being asked a *different* question from the set of three possible questions, how often will their answers happen to be the same (both Yes or both No)? If the plan is of type (a) or (d), both answers will always be the same. If the plan is of type (b) or (c), both answers will be the same 1/3 of the time. We conclude that no matter what type of plan each pair of twins may follow, the mere fact that they are following a plan implies that, when each of them is asked a different randomly chosen question, they will both give the same answer (which might be Yes or No) at least 1/3 of the time. It is important to appreciate that one needs data from many pairs of twins to see this effect, and that the inequality holds even if each pair of twins freely chooses any plan they like.

This, then, is the Bell argument: strong locality (no causal influences travelling faster than light from one twin to the other) and determinism (each pair of twins follows a plan) implies a Bell inequality (3).

D. What if the twins violate the Bell inequality?

In real experiments, when performing the analogous experiment on photons, the Bell inequality is violated, showing that no strongly local and deterministic theory can explain the data (cross-hatched shading in Fig. 1).

Let us imagine the same thing happening in our analogy. Suppose that when we analyze our results for a large sample of twins, we find that in cases where each twin was asked a different question, only 1/4 of the time are their answers the same; 3/4 of the time one gives a Yes and the other a No. This result violates the Bell inequality (3), and tells us that a good fraction of the population of twins must have *not* followed any predefined plan when they answered the questions. How do we interpret this result?

Our goal was to see if there was any evidence that the twins were communicating with each other using telepathic influences that travel faster than light. Suppose we don't want to accept the idea of supraluminal influences. The fact that the twins always agree when they are both asked the same question, even when they are being interrogated at spacelike separated locations, could be explained away by assuming they were following a prearranged plan. But if their pattern of answers to different questions violates the Bell inequality then this shows that they can't be following a prearranged plan. When one twin answers the question posed to him, we would have to assume that he sticks to the plan if he is being asked the same question as his twin but if he is being asked a different question from his twin then, at least some fraction of the time, he deviates from the plan, changing his answer in such a way that it differs from the answer that his brother is giving, and thereby violates the Bell inequality. This is a violation of strong locality: some of the time one twin's behavior is influenced by what question was posed to his brother, an event which happened at a spacelike separation from him. We are forced to accept that some sort of supraluminal influence connects the twins.

IV. EPR AND BELL WITH PHOTONS

The testing of twins for telepathic abilities, as described in section III, is an exact analogy to the EPRB experiment, which is a modification, suggested by Bohm [10], of the original EPR experiment. In the EPRB experiment (see Fig. 4) there is a source that creates pairs of photons, analogous to twins. The photons travel out from the source in opposite directions. When they are far from each other, each photon encounters a measuring machine that can do three possible measurements. The machine contains three types of filter, call them A,B, and C, and when the photon arrives the machine flips one of the three types of filter into the path of the photon. The photon has two possible responses to the filter: it either goes through the filter ("+") or reflects off it ("−"). This is actually a measurement of the polarization of the photon: each filter consists of polaroid with a different orientation. If determinism is true then each photon has an inherent polarization that determines how it will interact with each filter.

The EPRB experiment is therefore exactly analogous to the situation described in section III, where each twin gave a yes-or-no answer and there were three possible questions he might be asked; in that case determinism corresponded to the twins having a plan that determined how they would answer each question.

As with the twins, we can immediately see two ways to explain this consistent agreement.

1. *Causal influence*: when one photon reaches its machine and the machine decides what filter to flip up in front of it and the photon "decides" how to respond to that filter, those decisions somehow influence the other photon so that if the other photon gets the same filter, it will behave the same way.

2. *Determinism (a plan)*: when the photons are created, each is formed in a state (its "polarization state") that determines how it will respond to any possible filter it might encounter. The source puts both photons into the same state, ensuring that they always behave the same way when they encounter the same filter.

There are various additional assumptions being made here, such as:
• The measurement of each photon yields a unique result; this is called "macro-realism" in Ref. [1].
• The experimenters can decide what measurement to make without being controlled by the processes that determined the state of the photons; this is often called "lambda-independence" [11].
• The detector settings cannot exert causal influence backwards in time to affect the state of the photon.
We will assume the validity of these; for more details about them see Refs. [1, 9, 11].

The EPR argument (vertical shading in Fig. 1) is that in any strongly local theory it is not possible for causal influences to travel from one measurement to the other, so the agreement in same filter (same axis) measurements must arise from the photons having an internal state that determines their response to the filters that they encounter. If, as EPR did, one takes strong locality to be valid, then this shows that the photons are in a state that determines their behavior, which is in contradiction with the quantum mechanical picture where their state does not determine the outcome of measurements performed on them.

However, just as for the twins, there is a Bell argument (cross-hatched shading in Fig. 1) which shows that EPR's picture, of physical objects having definite states and strongly local interactions, can be experimentally tested. For this we look at the data for the cases when the two measuring machines deploy *different* filters in front

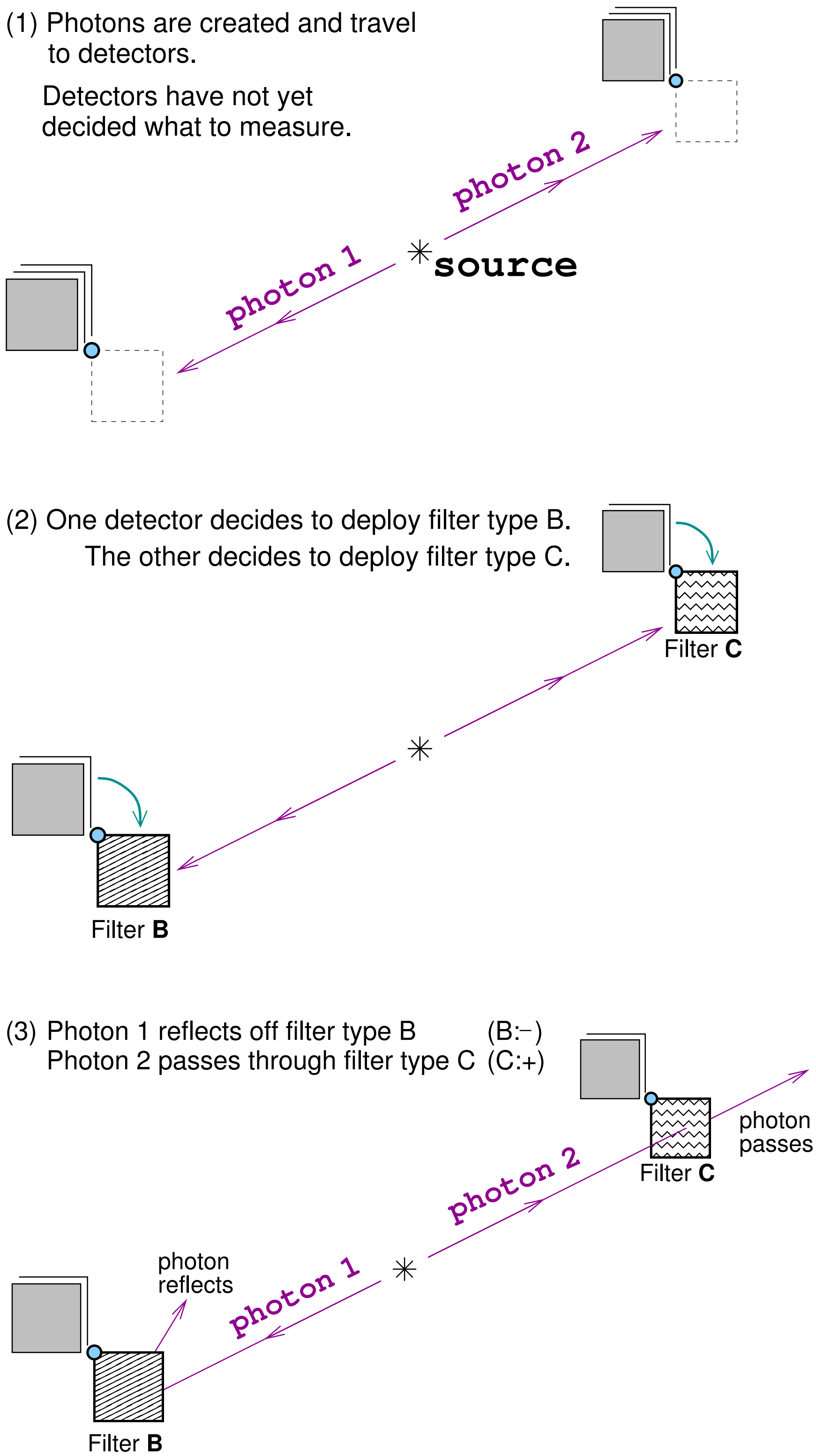

FIG. 4: One trial in the EPRB measurement of polarization for two photons. The final result in this trial is $(B : -, C : +)$, meaning that photon 1 encountered a filter of type B and reflected off it, while photon 2 encountered a filter of type C and passed through it. According to the Bell inequality (Eq. (4)) this sort of result, where the two photons do different things when encountering different filters, should happen no more than 2/3 of the time.

of the two photons. Following the logic used in Sec. III, we conclude that if both photons are in the same definite state, and there is no causal influence between their measurements, then, on the occasions that the detectors deploy *different* filters, then photons 1 and 2 should show the same behavior (both bouncing off or both passing through) at least 1/3 of the time:

Bell inequality:

$$\text{prob}\left(\begin{array}{l}\text{when photon 1 and photon 2}\\ \text{encounter different filters,}\\ \text{they show the same behavior}\end{array}\right) \geqslant 1/3 \qquad (4)$$

In Appendix A we show how Eq. (4) is a form of Bell's original inequality.

When polarizations of pairs of spin-singlet photons are measured in real-world experiments, it is found that they do show agreement in same-axis measurements, but when we perform different-axis measurements the two photons only show the same behavior 1/4 of the time; 3/4 of the time they show different behavior: one bounces off its filter and the other passes through (e.g. [12, 13]). This violates the Bell inequality. We conclude that strong locality is violated by spin-singlet photon pairs. Either you need a faster-than-light causal influence to make the same-axis agreement happen, or, if you try to save strong locality by using definite states instead, then you need a faster-than-light causal influence to obtain the observed violation of Bell's inequality for different-axis measurements.

V. CONSEQUENCES FOR LOCALITY

The EPRB experiment has provided us with strong evidence that nature does not obey the principle of strong locality: there are causal influences that travel faster then light. But this cannot be the end of the story. Perhaps it was asking too much to expect that *all* causal influences would be restricted to travel no faster than light, but in nature there certainly seems to be some limit on how fast some things can travel. Surely there must be some other weaker principle that expresses this feature of the natural world, and is compatible with Einstein's theory of relativity. We will now discuss what the possibilities are, and at what price a reasonable form of locality can co-exist with the EPRB experiment.

A. Different forms of locality

Physicists often think of locality as a single principle, but there are various requirements that can be thought of as expressing locality. Here are some of them along with a summary of how compatible they are with EPRB experiment results:

1) *Strong locality* (or *local causality*): all causal influences must travel slower than light.
As we have seen, this is disproven by EPRB experiments.
2) *Information* must be transmitted no faster than light.
This is also disproven by EPRB experiments, since the result of the measurement on one photon contains information about the measurement performed on the other that did not come from the backward light cone.
3) *Signal locality*: signals can travel no faster than light.
This is compatible with EPRB experiments, but at a price, as we will describe below.
4) *Energy or other conserved quantities* must travel no faster than light.
This is compatible with EPRB experiments, since no physical substance travels from one photon's measurement site to the other's.
5) *The Principle of Relativity*: The laws of physics are the same for any observer who is not accelerating (any "inertial frame of reference").
The tension between this principle and the EPRB experiments will be discussed below.

In the list above, the principle of signal locality stands out as a very natural alternative to strong locality. It is weaker than strong locality (not all causal influences are signals) but retains much of what we hope for in a locality principle, and, as we will see below, it is compatible with EPRB experimental results if we accept some "slipperiness" of nature, some limit on the controllability, accessibility, or definiteness of the state of things.

In Fig. 5 we show an augmented version of the simple theory-space diagram (Fig. 1), including the set of theories that obey signal locality (enclosed by the dashed (blue) line). If signal locality is valid then physics is restricted to the shaded (gold) area of allowed theories. These all violate strong locality, as required by the EPRB experimental results, and also depict nature as having an elusive, ungraspable character: the allowed theories are indeterministic (e.g. quantum mechanics), or deterministic but with sufficient uncontrollability of the hidden variables to prevent their being used to send faster-than-light signals.

B. EPR and Signal locality

In discussing signals, the essential point is that signalling is more than the transfer of information. Sending a signal means having a *controllable* means of transferring information. It may seem a little odd to invoke a concept like control, which is based on high-level entities with agency and free will, as part of a fundamental physical principle. Bell complained that signal locality "rests on concepts which are desperately vague, or vaguely applicable. The assertion that 'we cannot signal faster than light' immediately provokes the question: Who do we think *we* are?" [14].

Putting aside that concern, signal locality is plausible because it follows from the Principle of Relativity (no preferred inertial frame of reference) combined with the

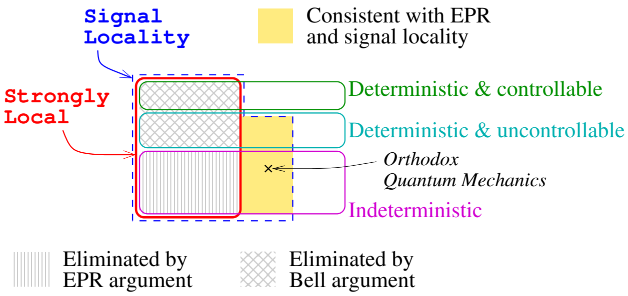

FIG. 5: Augmented version of Fig. 1, showing the set of theories obeying signal locality (enclosed by the dashed (blue) line), and dividing deterministic theories into those where the hidden variables can be controlled and those where they cannot. The shaded (gold) region is the set of theories where the indeterminism/uncontrollability is of the right kind to be consistent with EPRB experiments and signal locality. For details see Sec. V B.

requirement that we avoid causal paradoxes [9]. A causal paradox will arise if someone can send a signal to themselves in the past, since the person could, after receiving the signal, decide not to send it. To make sure that this cannot happen, we have to ensure that the sender of controllable information (signals) is always in the past of the receiver. In a theory that obeys the Principle of Relativity, this means that signals must go slower than light, since only then will all reference frames agree on the time ordering of the sender and receiver.

We can now see that EPRB experiments, which show that information can be transferred faster than light, also require some accompanying element of uncontrollability in nature in order to make sure that this does not translate to a violation of signal locality. We can imagine two ways to preserve signal locality in the face of the results of EPRB-type experiments.

(a) *Nature is indeterministic.*
The measurement outcomes are uncontrollable because nature is fundamentally indeterministic. In an indeterministic theory the results of measurements are "made up on the spot" when the measurement is done, and we have to rely on some sort of faster-than light influence to produce the consistent agreement in the results of same-axis measurements and also to produce the different-axis correlations that violate the Bell inequality. But this does not allow superluminal signalling because the measurement results are not determined by definite states of the objects in question: they are inherently random and therefore uncontrollable.

This is how standard formulations of quantum mechanics explain the correlations in an EPRB experiment. The collapse of the wavefunction is the faster-than-light influence. The information that is transferred from one measurement location to the other is information about the result of the measurement which arises randomly from the collapse of the wavefunction. Since it is uncontrollable there is no problem with this information being transferred faster than light. It cannot be used to send a signal.

(b) *Nature is deterministic but uncontrollable.* The measurement outcomes are uncontrollable because, even though they are determined by the states of the objects in question ("hidden variables"), those states are themselves sufficiently uncontrollable, because of some physical law, that they cannot be used to send signals. In such a scenario the violation of Bell's inequality in the different-axis measurements arises from a faster-than-light influence that allows one measurement to affect the hidden variables of the object being measured at the other, space-like separated, location, but the experimenter at one location cannot control the hidden variable states well enough to be able to control the measurement results at the other end, and thereby send a message. Bohmian Mechanics is an example of a theory that follows this pattern [9, 11]. Experimental inaccessibility of the hidden variables may also be an important factor in ensuring that they cannot be used for supraluminal signalling, but to the degree that they have faster-than-light effects on the results of remote measurements, they must be uncontrollable.

In Appendix B we describe a more formal way of exposing this dichotomy, by analyzing strong locality into two weaker conditions. Violation of one of them ("Remote Outcome Independence") corresponds to possibility (a) above; violation of the other ("Remote Detector Independence") corresponds to possibility (b).

C. EPR and the Principle of Relativity

We finally come to the question of how EPRB experiments cohere with the Principle of Relativity. To quote Bell himself, *"one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics—the real problems of quantum mechanics—they must be talking about Lorentz invariance"* [15]. In this quote, "Lorentz invariance" is just the Principle of Relativity, which states that the laws of physics are the same in all inertial reference frames, so the laws of physics are invariant under the Lorentz transformations that relate different reference frames to each other.

Bell's concern can be understood as follows. As we saw above, if we want to avoid causal paradoxes then the Principle of Relativity requires that signals travel no faster than light. In terms of Fig. 5 this means that the set of theories compatible with the Principle of Relativity is a subset of the set that obey signal locality. Bell's worry is that the Principle of Relativity might turn out to imply strong locality, in which case the Relativistic theories would be a subset of the strongly local ones, and EPR experiments would actually refute relativity itself [16]! So, is the observed uncontrollable faster-than-light causal connection between distant photons compatible with the Principle of Relativity? There is evidence that they are compatible, but not in the straightforward way that one might assume.

Naively one might say that of course the EPRB results are consistent with the Principle of Relativity, because they agree with the predictions of quantum mechanics, and there is a relativistic, Lorentz-invariant, formulation of quantum mechanics, namely quantum field theory. This might seem like an existence proof that the EPRB results are compatible with Relativity. The problem with this argument is that quantum field theory does not include the measurement postulate (wavefunction collapse induced by the measurement process) which is how the standard version of quantum mechanics explains how unique experimental results arise from measurements. There is no Lorentz-invariant version of measurement-induced wavefunction collapse that is compatible with the EPRB results [9]. However, this does not rule out the possibility that there may be other Lorentz-invariant theories that can explain the EPRB results. In fact, in 2006 an example was proposed: a version of quantum mechanics where the wavefunction occasionally collapses spontaneously in a Lorentz-invariant way [17]. Whether or not this theory is valid, it seems to provide an existence proof that EPRB results are compatible with the Principle of Relativity.

We should note that one can also take a different approach to Bell's question by abandoning measurement-induced wavefunction collapse and assuming that there is no collapse at all. This means dropping our "macro-realism" assumption (Sec. IV) that experiments have unique outcomes, and adopting something like the many-worlds [18] or many-minds [19] interpretation of quantum mechanics. One must then instead grapple with other questions, such as how probabilistic predictions emerge and the role of decoherence [11].

VI. SUMMARY

The EPRB experiment uses spin-singlet photon pairs to test the degree to which the laws of nature obey some sort of principle of locality. By separately measuring the spins of the two photons when they are spacelike separated, so only faster-than-light influences could allow the measurement of one photon to affect the measurement of the other, we find
— *same-axis agreement*: when we measure the spins of both photons along the same axis, the two results are always the same.
— *Bell-violating levels of different-axis agreement*: when we measure the spins of both photons along different axes, they agree less often than they would if the result were determined by some internal state of each photon.

If we accept some auxiliary assumptions (experiments have definite results; experimenters have some reasonable degree of autonomy; there is no backwards-in-time causality) then the EPRB experimental results bring us to the following conclusions.

- The observed behavior violates the strongest form of locality, which states that no causal influence can travel faster than light. In a nutshell, this is because either you need a faster-than-light causal influence to make the same-axis agreement happen, or, if you try to save strong locality by assuming that the agreement arises from the photons being in states that determine in advance that their spins will have specific values, then you need a faster-than-light causal influence to obtain the violation of Bell's inequality for different-axis measurements.

- In order to avoid casual paradoxes we expect nature to obey the principle of *signal locality* (signals cannot travel faster than light). This is compatible with the EPRB violation of strong locality as long as there remains some degree of elusiveness or ungraspability in nature. Either nature is indeterministic, so the results of the photon spin measurements are fundamentally random (as in the theory of quantum mechanics), or nature is deterministic, with the results of the measurements being determined by hidden variables, but there are fundamental limits on our ability to control those variables.

- The EPRB results are believed to be compatible with the Principle of Relativity (equivalence of all inertial reference frames, also called Lorentz Invariance), but standard quantum mechanics, with measurement-induced collapse of the wavefunction, is not. The collapse itself violates Lorentz Invariance. However, there exists a modification of quantum mechanics in which the wavefunction undergoes random spontaneous collapse in a Lorentz-invariant fashion. The existence of this theory, which correctly predicts the outcome of EPRB experiments, indicates that the EPRB results are compatible with Lorentz invariance of the laws of nature.

VII. CONCLUSIONS

The EPRB results lead us to accept the existence of some sort of faster-than-light causal influence in nature, but this influence is of a very unusual kind. It is not attenuated at all with distance (two photons in a spin-singlet state can be arbitrarily far apart and still show the EPRB correlations) and it only connects specific particles that were created in an entangled states, like the spin-singlet for photon pairs. Other particles that are not part of the entangled state are not affected. Quantum mechanics explains these phenomena in terms of the wavefunction which is a full specification of the state of a system, but only a probabilistic specification of the outcome of measurements. The outcome of measurements contains a fundamental element of randomness.

The EPRB results provide a challenge to any attempt to replace quantum mechanics with a more traditional theory in which there are additional variables whose state determines the outcomes of measurements. We see that in such a theory nature would still have an elusive, ungraspable character. The theory would have to incorporate fundamental limits on our control over the additional variables in order to avoid faster-than-light signalling and the attendant causal paradoxes. To many people, including some physicists, this might seem a price worth paying. There is still discomfort with orthodox quantum mechanics, in which the wavefunction has physical reality as a full specification of the state of a system, but its time evolution according to the Schrödinger Equation is occasionally and arbitrarily interrupted by measurement-induced collapse for which there is no clear governing law and which violates Lorentz invariance (admittedly without fatal consequences such as superluminal signalling). In spite of the EPRB experiment's illumination of the obstacles, the search for a more satisfactory account of the microscopic workings of nature seems destined to continue.

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Appendix A: Relationship to Bell's original inequality

Here we describe how our informally derived inequality Eq. (4) follows from the mathematical inequality derived by Bell. Let us define

$$p(+-|AB) \equiv \text{probability that, \textit{given} that machines 1 and 2 have decided to deploy filters A and B respectively, photon 1 passes through and photon 2 bounces off} \tag{A1}$$

and so on. Then Bell's original inequality is

$$\begin{aligned} p(+-|AB) + p(+-|BC) + p(+-|CA) &\leqslant 1 \\ \text{or, equivalently,} & \\ p(-+|AB) + p(-+|BC) + p(-+|CA) &\leqslant 1\ . \end{aligned} \tag{A2}$$

To derive our inequality (4) from Bell's original inequality, start by rewriting (4) as

$$p_{\rm diff} \leqslant 2/3$$

where

$$p_{\rm diff} = {\rm p}\left(\begin{array}{l}\text{when photon 1 and photon 2}\\ \text{encounter different filters,}\\ \text{they show different behavior}\end{array}\right) , \tag{A3}$$

then using the notation (A1) and defining $p(AB)$ to be the probability that machine 1 deploys the A filter and machine 2 deploys the B filter, and so on, we can rewrite $p_{\rm diff}$ as a sum over all filter settings $F = (AB, BC, CA, BA, CB, AC)$ where the two detectors deploy different filters:

$$p_{\rm diff} = \frac{\sum_F p(F)\Big(p(+-|F) + p(-+|F)\Big)}{\sum_F p(F)} . \tag{A4}$$

In our experiment, each filter is deployed at random, so all six combinations occur with equal probability,

$$p_{\rm diff} = \frac{1}{6}\sum_F \Big(p(+-|F) + p(-+|F)\Big) . \tag{A5}$$

The labeling of photons and measuring machines as "1" and "2" is arbitrary, so with no loss of generality we can treat the BA filter deployment as being AB with the numbering of the photons and machines reversed, so $p(+-|AB) = p(-+|BA)$ and so on, so (A5) can be written

$$\begin{aligned} p_{\rm diff} = \frac{1}{3}\Big(& \; p(+-|AB) + p(+-|BC) + p(+-|CA) \\ + & \; p(-+|AB) + p(-+|BC) + p(-+|CA)\Big) . \end{aligned} \tag{A6}$$

Using Bell's original inequality (A2) we recover our inequality (A3).

Appendix B: Remote Detector vs. Remote Outcome independence

As we saw in Sec. V B, there are two forms of non-controllability that would allow us to keep the desirable principle of signal locality in the face of the EPR experiment's results. Jarrett [6] pointed out one way to approach this. As illustrated in Fig. 6, strong locality can be written as a combination of two requirements on the outcome of measurements at a given detector.

Remote Detector Independence *and* Remote Outcome Independence ⇒ Strong Locality

⇑

Determinism

FIG. 6: Strong locality can be understood as saying that the outcome at one detector is independent of both the settings of the remote detector, and the outcome of the remote measurement.

1. *Remote Detector independence*, more commonly known as *Parameter independence*: the outcome of the measurement of one photon is not affected by the detector setting for the other photon, but may be affected by the measurement outcome of the other photon. As we will describe below, this form of locality is compatible with EPR experiments and signal locality, but requires us to give up determinism.

2. *Remote Outcome independence*: the outcome of the measurement of one photon is not affected by the outcome of the remote measurement, but may be affected by the remote detector setting. This allows us to keep determinism, but requires some limits on our ability to control the hidden variables whose states determine measurement outcomes.

We can then understand EPR experiments as requiring us to accept that one of these conditions must be violated, but the other may be valid.

(a) Remote Outcome independence is violated. This requires us to give up determinism because Remote Outcome independence can be shown to follow from determinism [1, 6], so if Remote Outcome independence is violated then determinism must be as well. This corresponds to possibility (a) in Sec. V B. We can save signal locality because without determinism there is the possibility of randomness (and hence uncontrollability) in measurement outcomes. Some faster-than-light influence allows the outcome of one measurement to influence the other, and this is the mechanism that explains both the consistent agreement of same-axis measurements and the violation of the Bell inequality in different-axis measurements. Quantum Mechanics is an example of a theory that violates Remote Outcome independence while preserving Remote Detector independence.

(b) Remote Detector independence is violated. Allowing measurement results to be influenced by the settings of the remote detector allows us to keep determinism, so there are hidden variables whose state determines the outcome of measurements, but to keep signal locality we have to allow essential limits on our ability to control the state of the hidden variables. This corresponds to (b) in Sec. V B. Bohmian Mechanics is an example of a theory that, *pace* Dickson [20], is usually said to violate Remote Detector independence while preserving Remote Outcome independence [9].